ISSN 2395-1621

Vibration Analysis and Effect of Impact Loading on Cantilever Beam

[#1]Mr. Raviraj S. Bhosale, [#2]Mr. Satish M. Margutti, [#3]Mr. ShrinivasV.Shelge

[1]ravirajbhsale43@gmail.com

[2]s.margutti@gmail.com

[3]shelgevpcoe@gmail.com

[#1]Post graduate Student (Mechanical design), Vidya Pratishtan's College of Engineering, Baramati.

[#2]Assistant Professor, Department of Mechanical Engineering, Vidya Pratishtan's College of Engineering, Baramati.

[#3]Assistant Professor, Department of Mechanical Engineering, Vidya Pratishtan's College of Engineering, Baramati.

ABSTRACT

Vibration failure is the predominant mode of structural elements. Crack initiation start with respect to time, so it is necessary to evaluate the crack location. The dynamic behavior of a whole structure is affected due to the presence of a crack as the stiffness of that structural element is replaced. Frequency change due to crack observed in structure, amplitudes of free vibration and dynamic stability areas to a foreseeable extent. Experiment performed on aluminum beam with different crack location and crack angles. There are two cracks on aluminum beam which are 240mm and 540mm long from starting position. Similarly crack angles found $0^0,15^0,30^0,45^0,90^0$. In this paper free and forced vibration analysis of cracked beam is performed in order to identify vibration analysis of cantilever beam. Obtained results shows that, for constant crack depth and variable crack location there is increment in natural frequency found at second mode and decrement found at third mode. It is found that at first mode there is no appreciable variation in natural frequency. Impact load height is a considerable parameter described in this paper. As impact load height increases stresses also increases in cracked beam as compare to uncracked beam. Experimentally it is found that, crack value increases from opening to closing of the crack. ANSYS 14.5 software is used for simulation work. It is found that well match of experimental and numerical result.

Keywords— **Vibration Analysis, Crack, Beam, ANSYS 14.5, Natural Frequency.**

ARTICLE INFO

Article History

Received :18th November 2015

Received in revised form :

19th November 2015

Accepted : 21st November , 2015

Published online :

22nd November 2015

I. INTRODUCTION

For the last several years, a considerable amount of research work has been studied to introduce the faults in structures. It has been observed that most of the mechanical elements fail due to the present of cracks. Cracks are present in the structure due to different types of reasons. The cracks are developed mainly due to fatigue loading. Therefore the detection of cracks and vibration analysis is an important aspect of structural design. Cracks are generally occurring due to stresses present in structural elements.

Damage is defined as any deviation introduced to a structure, either purposely or unintentionally, which adversely affect the current or future performance of that system. The presence of a crack could not only cause a local variation in the stiffness but it could affect the mechanical behavior of the entire structure to a considerable amount.

Cracks may be occurred by fatigue under service conditions as a result of the limited fatigue life. They may also occur due to mechanical impurity and defects.

The most common structural defect is the existence of a crack. Cracks are among the most encountered damage types in the structure due to fatigue or manufacturing defects. Crack will start in a structure when the stresses near the crack tip will exceed the permissible limit. Cracks occurred in structural elements may arise due to fatigue cracks that take place under service conditions as a result of the limited fatigue strength. Cracks may also occur mechanical defects or defects

due to manufacturing processes. Mechanical accidents, fatigue, erosion, as well as environmental effects, are issues that can lead to a crack in a mechanical structure or elements. Generally cracks are tiny in shapes. Such tiny cracks are known to broadcast due to different stress conditions. If these propagating cracks remain undetected and reach their critical size, then a sudden structural failure may observe.

Beams are one of the most commonly used structural elements in numerous engineering applications and experience a wide variety of static and cyclic loads. Cracks may built in beam-like structures due to such loads or forces. Considering the crack as a important form of such damage, its modeling is an significant step in studying the behavior of break down structures. Knowing the effect of crack on stiffness, the beam or shaft can be modeled using either Euler-Bernoulli or Timoshenko beam theories. The beam border conditions are used along with the crack compatibility relations to derive the characteristic equation relating the natural frequency, the crack depth, crack length, crack angle and location with the other beam properties.

However, if the load applied to a structure is suddenly applied then one will observe that such dynamically applied load will create higher stresses, compare to statically applied load for a short time. We need to know the effect of suddenly applied loads such as earthquake, collision between cars or trains which create dynamic loads and hence higher damage takes place than statically applied loads.

II. AIM AND OBJECTIVE

In the present research, a number of papers published thus far have surveyed, reviewed and analyzed. Most of the researchers studied the effect of a single crack on the dynamics of structures. However, in actual structural members such as beams, shafts are highly susceptible to transverse cross-sectional cracks due to fatigue. Some information is available on dynamics of structures due to crack but this is not exhaustive for real applications. For that purpose in this analysis we consider vibration analysis and effect of dynamic load on cracked beam. In this work we consider the cracked cantilever beams of a relatively large aspect ratio. Due to large aspect ratio bending mode is more significant compared with the shear deformation of the beam especially for the long pipelines; railway tracks etc. As a damage mode, cracks on the beam could take various formats, such as transverse crack, longitudinal cracks, open cracks, surface or internal cracks, slant cracks, regular or irregular cracks, or a addition of any of these.

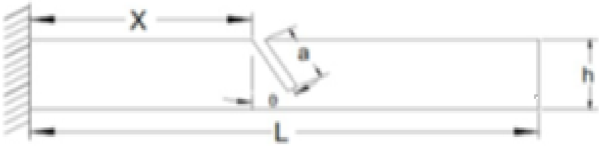

Fig.1: Slant Cracks

The objective of this work is to analyze experimentally and numerically the vibration analysis on the slant cracked cantilever beam and effect of impact load on the slant cracked cantilever beam. To formulate all the data related to this work. To measure the effect of impact load on stresses in a beam with experimentally and numerically.

The steps of the process plan for the present work are as follows;

1. Vibration analysis of cantilever beam.
2. With the help of finite element method (Numerically), we find out the first three modal natural frequencies.
3. Experimental Analysis to obtain the Relative values of first, second and third modal natural frequencies, by performing the laboratory experiment for beam.
4. To measure the effects of impact load on stresses in a beam.
5. With the help of finite element method (Numerically), we find out the stresses in the cracked beam.

A) Methodology

In this case, it is necessary to measure or compute the first three transverse natural frequencies of the beam with a crack and the corresponding uncracked beam. As stated earlier, both the crack location and the crack depth influence the changes in the natural frequencies of a cracked beam. accordingly, a exacting frequency could correspond to different crack locations and crack depths. In our research work we consider Aluminum beam with Slant crack. The steps of research methodology are as follows:

1. In the First stage of the work slant cracks are included for developing the analytical expressions on dynamic characteristics of structures.
2. With the help of ANSYS calculate the first three natural frequencies which are related to bending mode for cracked as well as uncracked beam.
3. To experimentally validate cases of the three natural frequencies of cracked as well as uncracked beam experimental modal analysis will be done using FFT analyzer.
4. By comparing first three natural frequencies obtained from ANSYS and experimental results, normalized frequencies are obtained for the case with minimum error.
5. To measure the effects of impact load on stresses in a beam.
6. With the help of finite element method (Numerically), we find out the stresses in the cracked beam.

In the last stage of the investigation the effect of crack depth and crack location on the modal values of natural frequencies are obtained with a very convincing manner. And the data obtained from the Numerical analysis and

Experimental analysis are compared. Suitable numerical methods are used in sequence to solve the theoretical equations developed. We also find out the effect of dynamic load on cracked beam and uncracked beam. Useful conclusions are drawn from the numerical results of respective parts. The numerical results are validated by using the experiments on beams.

III. EXPERIMENTAL MODAL ANALYSIS

A) Introduction

Experimental modal analysis has grown steadily in popularity since the advent of the digital FFT spectrum analyzer in the early 1970s. Today, impact testing has become widespread as a fast and economical means of finding the modes of vibration of a machine or structure. Modal testing and experimental modal analysis is the process of characterizing the dynamic properties of a test structure by exciting the structure artificially and identifying its modes of vibration. When a structure is damaged e.g. its geometrical properties change, its boundary conditions modify or its material properties alter the dynamic characteristic of the structure change. These changes are the basic for the damage identification methods and it serves as a means to extract dynamic properties for a given structure.

B) Experimental Work

a) Specimen Geometry

Aluminum beams have been considered for making specimens. The specimens were selected of size having 25mm* 16mm cross sectional area. The specimens were cut to size from ready-to-wear rectangular bars. Total 6 specimens were cut to the size of length 500mm and cross section area as 25mm*1mm. for experimentation work first we consider uncracked beam as a base line model and remaining six beam with crack having 5 mm depth total six beams cracked at specified location and having specified crack depths and which are used for experimental modal analysis.

Material Properties of the aluminum material which is used vibration analysis. Which is given below,

Material- Aluminum
Grade- 63400(HE9) WP
Width of Beam- 25mm
Height of Beam- 16mm
Length of Beam- 740mm
Modulus of elasticity- 73GPa
Poisson ration- 0.33
Density- 2780 kg/m^3.

Different specimen for modal analysis:

Table I: Different specimen for modal analysis.

Case No.	Crack location "mm"	Crack Position "x^o"	Crack depth "mm"
1st	240 and 540	0	5
2nd	240 and 540	15	5
3rd	240 and 540	30	5
4th	240 and 540	45	5
5th	240 and 540	90	5

b) Experimental Set-up

In this experimentation the beam is fixed with the help of nut-bolts to I section beam support mounted on foundation, which acts as fixed support to create cantilever condition the experimental modal analysis of the beam is carried out to determine the natural frequencies of beam. The response of beam to harmonic excitation is obtained by vibration exciter, force transducer, accelerometer and vibration analyzer. Experimental set up consists of following components

1. Impact hammer.
2. Accelerometer.
3. FFT Analyzer.
4. Display unit (Computer system).
5. Specimen.

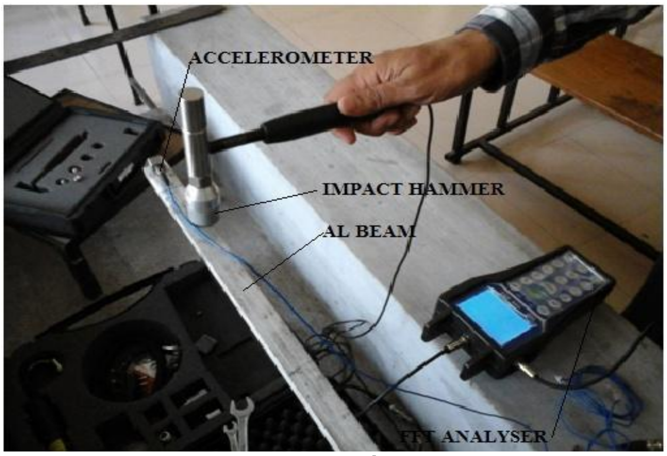

Fig.2: Experimental Set-up.

c) Experimental Procedure

To identify the dynamic properties of aluminum beams, modal testing was performed. In modal testing beams were excited by a modally tuned impact hammer at a certain reference point and responses of the beams were measured by an accelerometer. In experimental modal analysis, the renewed signals from the hammer and the accelerometers were analyzed and the modal parameters of the beams were determined. Data collection for all the damage cases and one healthy beam were done according to the procedure given below:

1. Fix the beam to the rigid support using nut and bolt so that beam is in horizontal position and crack is on upper side.
2. Fix the accelerometer with magnetic base on the specimen at specified location.
3. The beam was excited by an impact hammer and data from measurement point was collected with an accelerometer.
4. Accelerometer was used to measure the frequency response and read from a FFT Vibration Analyzer.
5. Response spectrum of beam gives the natural frequencies of beam.

The effects of a crack, made at different angles, on the first three modal frequencies have been determined in this project. Before the experiments were carried out, the first three natural frequencies of the beams were determined by FEA. The results of modal analysis done in ANSYS14.5 workbench for healthy and beams having cracks, first we have performed experiment on baseline model then cracked beam. After carrying out the experimentation following results were obtained.

Table II: Natural frequency of uncracked beam

Uncracked beam	First freq. HZ	Second freq. HZ	Third freq. Hz
1	23	37	149

Table III Natural frequency of cracked Beam

Crack angle(x^0)	First freq. HZ	Second freq. HZ	Third freq. Hz
0	23	38	149
15	22	38	150
30	23	39	149
45	23	38	148
90	23	39	149

IV. EFFECT OF IMPACT LOADING ON BEAM

We have studied the stresses caused by static loads on a structure. However, if the load applied to a structure is suddenly applied then one will observe that such dynamically applied load will create higher stresses, compare to statically applied load for a short time. We need to know the effect of suddenly applied loads such as earthquake, collision between cars or trains which create dynamic loads and hence higher damage takes place than statically applied loads in this experimentation work we have developed experimental set up based on electrical strain gauge method which works on following principles.

Fig.10: Experimental Set-up for Effect of Impact Loading

Table IV: Effect of impact loading on uncracked beam.

Crack Angle	Weight (gm)	Weight (N)	Height (mm)	strain
Uncracked	111	1.0889	100	$333e^{-6}$
Uncracked	111	1.0889	150	$370e^{-6}$
Uncracked	111	1.0889	200	$502e^{-6}$

Table V: Effect of impact loading on cracked beam (0 Degree)

Crack Angle	Weight (gm)	Weight (N)	Height (mm)	strain
0	111	1.0889	100	$335e^{-6}$
0	111	1.0889	150	$375e^{-6}$
0	111	1.0889	200	$505e^{-6}$

Table VI: Effect of impact loading on cracked beam (15 Degree)

Crack Angle	Weight (gm)	Weight (N)	Height (mm)	strain
15	111	1.0889	100	$355e^{-6}$
15	111	1.0889	150	$425e^{-6}$
15	111	1.0889	200	$518e^{-6}$

Table VII: Effect of impact loading on cracked beam (30 Degree)

Crack Angle	Weight (gm)	Weight (N)	Height (mm)	strain
30	111	1.0889	100	$342e^{-6}$
30	111	1.0889	150	$416e^{-6}$
30	111	1.0889	200	$511e^{-6}$

Table VIII: Effect of impact loading on cracked beam (45 Degree)

Crack Angle	Weight (gm)	Weight (N)	Height (mm)	strain
45	111	1.0889	100	$371e^{-6}$
45	111	1.0889	150	$465e^{-6}$
45	111	1.0889	200	$585e^{-6}$

Table IX: Effect of impact loading on cracked beam (90 Degree)

Crack Angle	Weight (gm)	Weight (N)	Height (mm)	strain
90	111	1.0889	100	$375e^{-6}$
90	111	1.0889	150	$530e^{-6}$
90	111	1.0889	200	$700e^{-6}$

Table X: Impact stresses in uncracked beam

Crack Angle	Weight (gm)	Weight (N)	Static stress (N/mm2)	Expt Impact stress(N/mm2)
Uncracked	111	1.0889	10.88	50.55
Uncracked	111	1.0889	10.88	60.12
Uncracked	111	1.0889	10.88	76.14

Table XI: Impact stresses in cracked beam (0 Degree)

Crack Angle	Weight (gm)	Weight (N)	static stress(N/mm2)	Expt Impact stress(N/mm2)
0	111	1.0889	10.88	50.55
0	111	1.0889	10.88	56.14
0	111	1.0889	10.88	75.48

Table XII: Impact stresses in cracked beam (15 Degree)

Crack Angle	Weight (gm)	Weight (N)	static stress(N/mm2)	Expt Impact stress(N/mm2)
15	111	1.0889	10.88	55.58
15	111	1.0889	10.88	65.072

15	111	1.0889	10.88	76.06

Table XIII: Impact stresses in cracked beam (30 Degree)

Crack Angle	Weight (gm)	Weight (N)	static stress(N/mm2)	Expt Impact stress(N/mm2)
30	111	1.0889	10.88	50.55
30	111	1.0889	10.88	62.46
30	111	1.0889	10.88	76.06

Table XIV: Impact stresses in cracked beam (45 Degree)

Crack Angle	Weight (gm)	Weight (N)	static stress(N/mm2)	Expt Impact stress(N/mm2)
45	111	1.0889	10.88	55.48
45	111	1.0889	10.88	69.75
45	111	1.0889	10.88	88.16

Table XV: Impact stresses in cracked beam (90 Degree)

Crack Angle	Weight (N)	Weight (N)	static stress(N/mm^2)	Expt Impact stress(N/mm^2)
90	111	1.0889	10.88	55.58
90	111	1.0889	10.88	77.16
90	111	1.0889	10.88	101.90

VI. FINITE ELEMENT ANALYSIS

The three natural frequencies are determined by the numerical method (FEM) using ANSYS version 14.5. First the aluminum cantilever of the dimensions is being modelled without crack and with crack at different positions. By modal analysis three natural frequencies are obtained. Following stepladder show the guidelines for carrying out Modal analysis.

1. Define Materials.
2. Set preferences. (Structural).
3. Define constant material properties.
4. Model the Geometry.
5. Follow bottom up modeling and create the geometry.
6. Generate Mesh.
7. Define element type.
8. Mesh the area.
9. Apply Boundary Conditions.
10. Apply constraints to the model.
11. Obtain Solution.
12. Specify analysis types and options.
13. Solve.

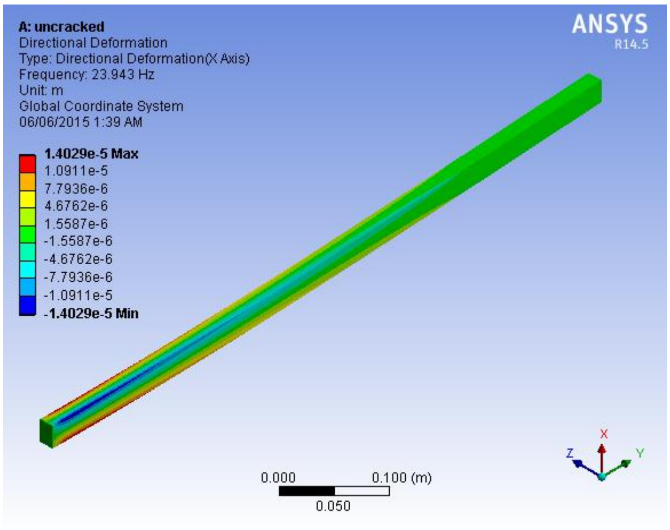

Fig.4: First frequency of uncracked beam.

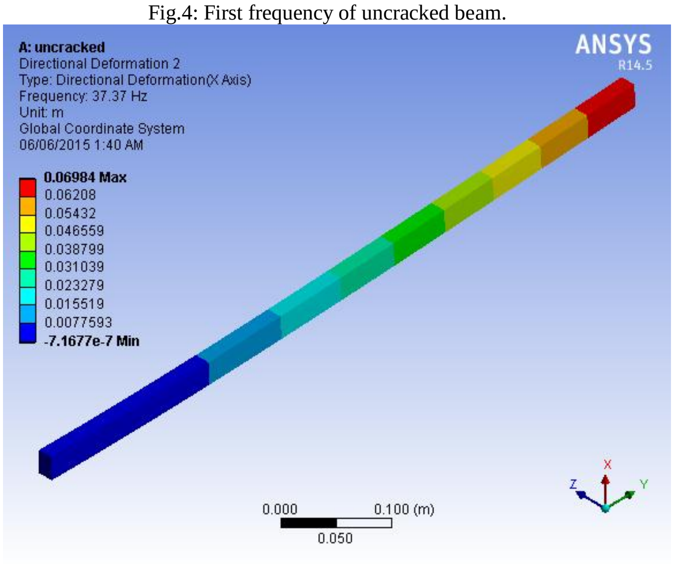

Fig.5: Second frequency of uncracked beam.

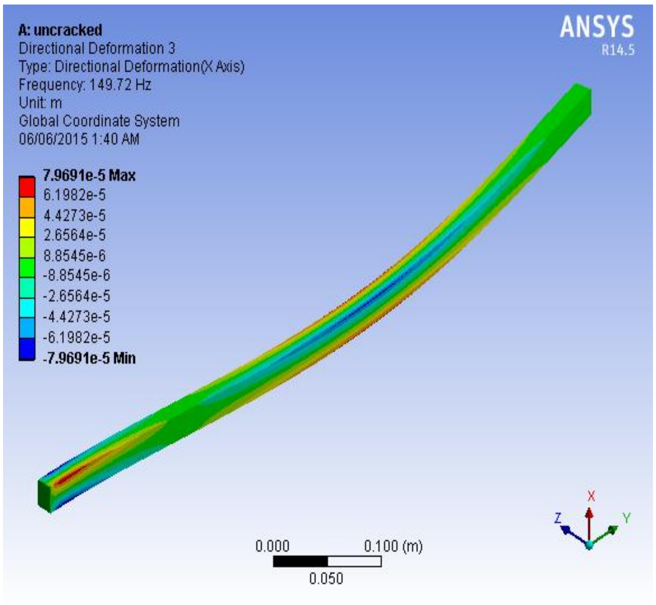

Fig.6: Third frequency of uncracked beam.

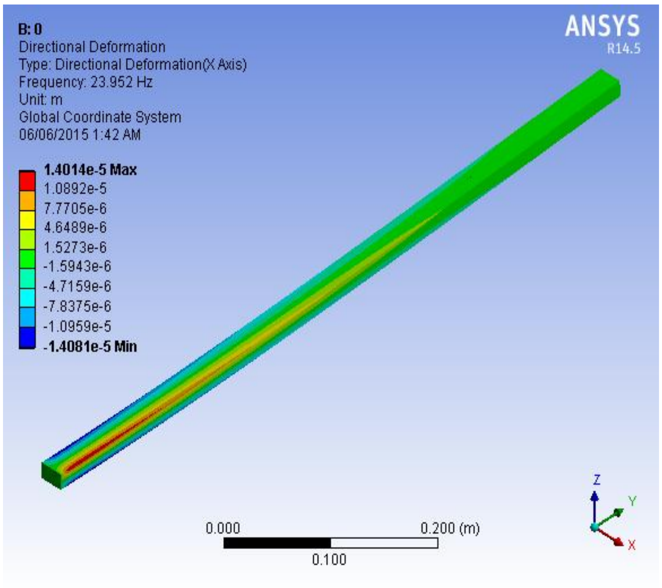

Fig.7: First frequency of 0 Degree beam.

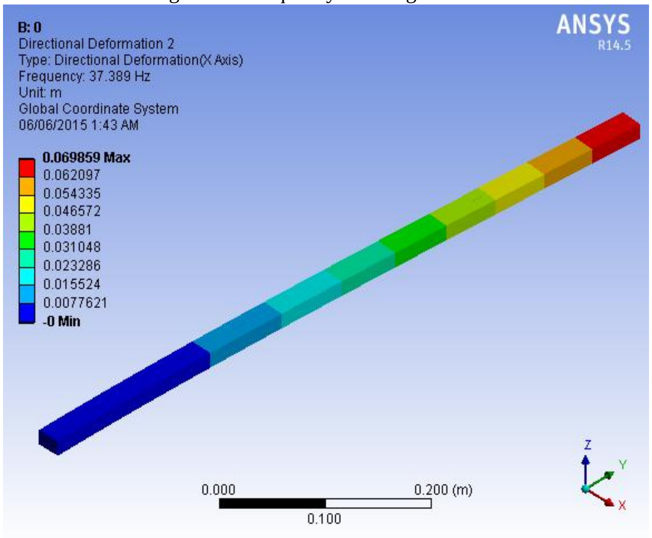

Fig.8:Second frequency of 0 Degree beam.

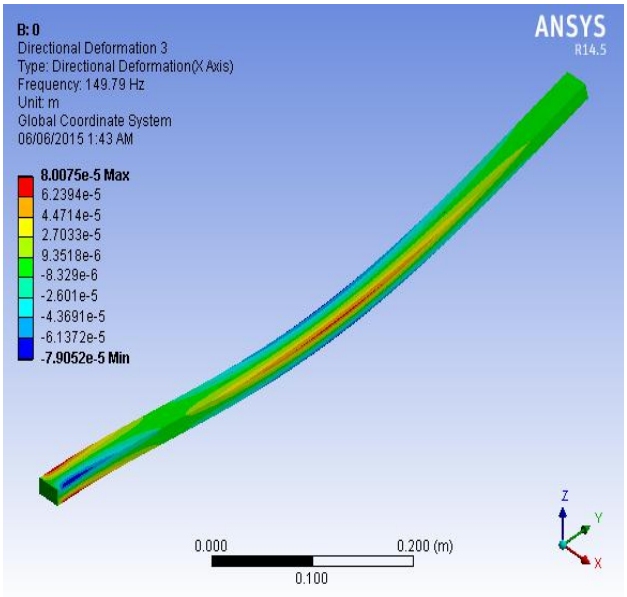

Fig.9: Third frequency of 0 Degree beam.

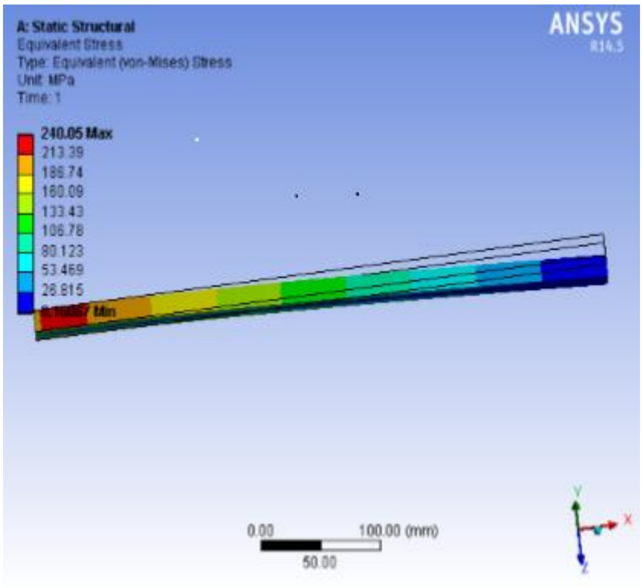

Fig. 10 Stresses in Uncracked beam

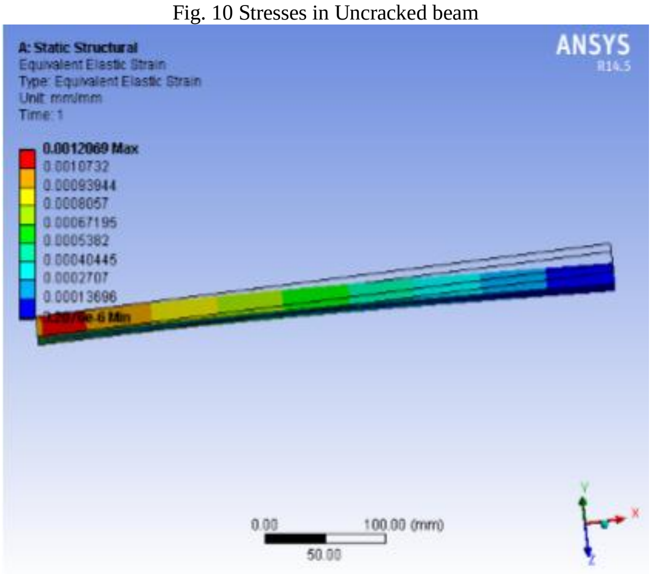

Fig.11: Strain value in Uncracked beam

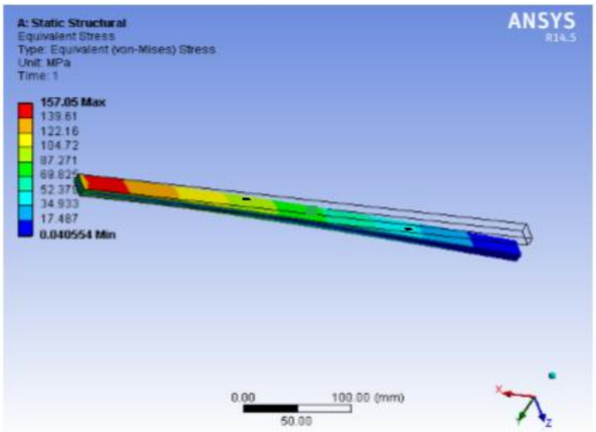

Fig. 12: Stresses in cracked beam at 0^0

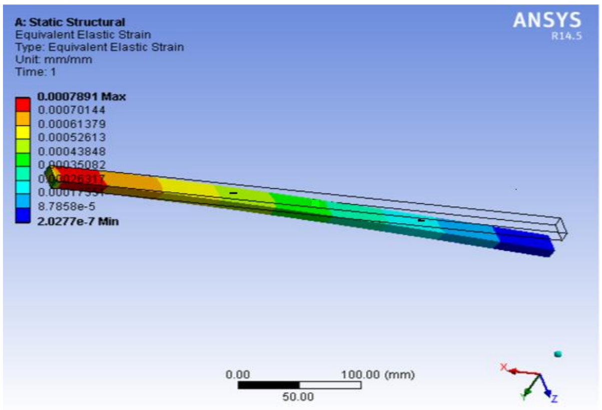

Fig.13: Strain value in cracked beam at 0^0

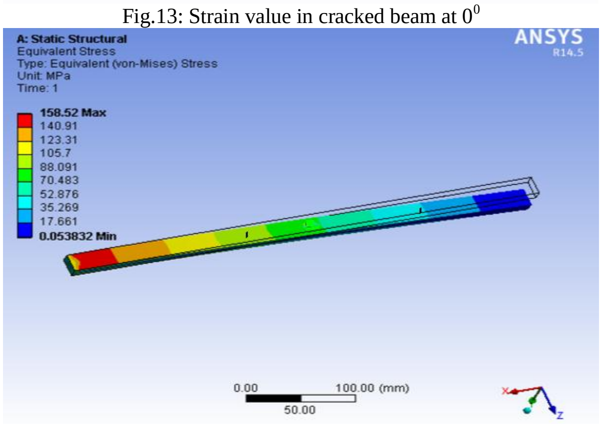

Fig. 14: Stresses in cracked beam at 30^0

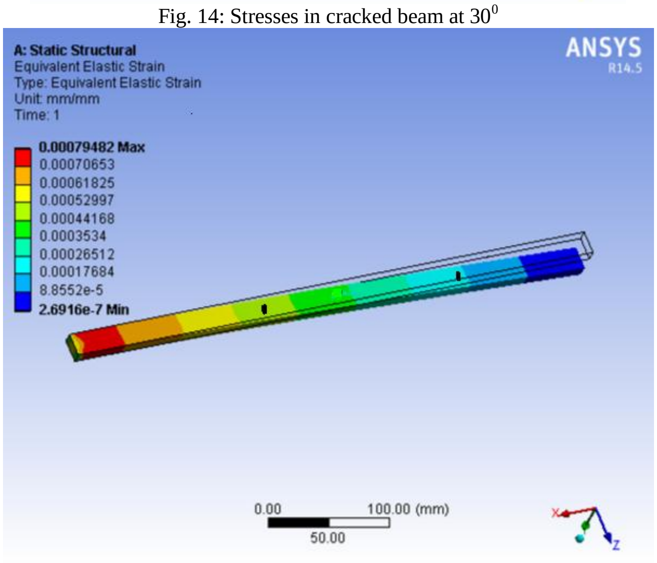

Fig.15: Strain value in cracked beam at 30^0

VI. RESULT AND DISCUSSION

The effects of a crack, present at different locations, on the first three modal frequencies have been determined in this project. Before the experiments were carried out, the first three natural frequencies of the beams were determined by FEA. The results of modal analysis done in ANSYS 14.5 Workbench for healthy and 7 cases of cracked beam are tabulated in Table XVI to XXI respectively. Table XVI shows the first three natural frequencies of healthy beam. Table XVII onwards shows the variation of the fundamental frequency as a function of the crack depth and crack location for the cracked beam. The results from experimental and numerical methods are compared in the tables and the mode shapes obtained from the numerical method are also shown in the above figure.

Table XVI: Natural frequency of Uncracked beam.

Mode No	Experimental (Hz)	Numerical (Hz)	Percentage Error
1	23	23.94	3.92
2	38	37.37	0.99
3	144	149.72	0.48

Table XVII: Natural frequency of cracked beam at 0^0

Mode No	Experimental (Hz)	Numerical (Hz)	Percentage Error
1	23	23.95	3.96
2	38	37.389	1.63
3	149	149.79	0.48

Table XVIII: Natural frequency of cracked beam at 15^0

Mode No	Experimental (Hz)	Numerical (Hz)	Percentage Error
1	22	23.88	7.87
2	38	37.382	1.65
3	150	149.32	0.45

Table XIX: Natural frequency of cracked beam at 30^0

Mode No	Experimental (Hz)	Numerical (Hz)	Percentage Error
1	23	23.85	3.56
2	39	37.389	4.30
3	149	149.80	0.53

Table XX: Natural frequency of cracked beam at 45^0

Mode No	Experimental (Hz)	Numerical (Hz)	Percentage Error
1	23	23.91	3.80
2	38	37.388	1.63
3	148	149.48	0.99

Table XXI: Natural frequency of cracked beam at 90^0

Mode No	Experimental (Hz)	Numerical (Hz)	Percentage Error
1	23	23.868	3.63
2	39	37.378	4.33
3	149	149.20	0.13

Table XXII: Comparison between Expt. and Numerical stress in Uncracked beam

Crack	Experimental Stress (N/mm^2)	Numerical stress (N/mm^2)	Percentage Error
Uncracked	50.55	56.23	10.10
Uncracked	60.12	66.20	9.81
Uncracked	76.14	85.17	10.60

Table XXIII: Comparison between Expt. and Numerical stress in cracked beam $\theta = 0^0$

Crack	Experimental Stress (N/mm^2)	Numerical stress (N/mm^2)	Percentage Error
0^0	50.55	57.16	11.56
0^0	56.14	64.5	12.96
0^0	75.48	79.80	5.41

Table XXV: Comparison between Expt. and Numerical stress in cracked beam θ= 15^0

Crack	Experimental Stress (N/mm^2)	Numerical stress (N/mm^2)	Percentage Error
15^0	55.58	60.37	7.93
15^0	65.072	59.50	8.56
15^0	76.06	73.46	3.41

Table XXVI: Comparison between Expt. and Numerical stress in cracked beam θ= 30^0

Crack	Experimental Stress (N/mm^2)	Numerical stress (N/mm^2)	Percentage Error
30^0	50.55	55.17	8.3
30^0	62.46	70.48	11.37
30^0	76.06	90	12.36

Table XXVII: Comparison between Expt. and Numerical stress in cracked beam θ= 45^0

Crack	Experimental Stress (N/mm^2)	Numerical stress (N/mm^2)	Percentage Error
45^0	55.48	59.18	6.25
45^0	69.75	74.63	6.53
45^0	88.16	91.93	4.10

Table XXVIII: Comparison between Expt. and Numerical stress in cracked beam θ= 90^0

Crack	Experimental Stress (N/mm^2)	Numerical stress (N/mm^2)	Percentage Error
90^0	55.58	58.92	5.66
90^0	77.16	87.92	12.23
90^0	101.90	123.38	15.40

VI. CONCLUSION

Mode shapes and natural frequencies of the vibrating structures are susceptible to change under the influence of crack position and crack orientation. Mode shapes in magnifying views allow the researchers to get an idea of the significant changes at the crack location. Therefore situation and strictness of crack can be determined by analyzing these changes. An extensive literature survey showed that there is little work done on effect of impact loading on beam. Most of the available articles on structural dynamic analysis and modal

analysis for transverse crack. None of the published articles work on slant crack. Results obtained through this study can be summarized as follows:

1. When the crack location is constant and crack depth is same but the crack position is different the natural frequency of the beam increases for all cracked beam in second mode but in third mode natural frequency goes on decreasing for $15^0$and 45^0. In first mode such type of crack there is no such more variation.
2. If Crack at an 30^0 that time frequency drastically goes on decreasing and which is more difference as compared to all other type of crack position.
3. When the crack location is constant and crack depth is same but the crack position is different that time the variation of amplitude is decreasing up to crack location and after that it goes on increasing up to its end.
4. If height goes on increasing for impact the stress value goes on increasing for all cracked beam as compared to uncracked beam. If crack at an 45^0 and 90^0 then the stress value suddenly goes on increasing. The stress value goes on increasing from crack opening to crack closing.

ACKNOWLEDGMENT

It gives us great pleasure to present this dissertation paper on "Vibration Analysis and Effect of Impact Loading on Cantilever Beam ". It has been great pleasure for me to work with Prof. S.V. Shelge and Prof. S. M. Margutti on this outstanding dissertation. His knowledge, diligence and innovativeness extended my desire to be a great engineer. I consider myself one of the luckiest people to be his student. I would like to thank to Prof. S.M. Margutti for not only providing me guidance throughout the research and in my coursework expands my academic vision.

I am sincerely thankful to Prof. P.V. Gogte for providing an enthusiasm to us, his students. It has been a great honor to be one of his students. Dissertation that I have done for him not only improved my ability to think critically but also improved my writing skill. I appreciate his concern for student's development. I am also grateful to P.G. Coordinator Prof. A.H. Kolekar for sparing his valuable time and supporting me to undertake my work.

REFERENCES

[1]K. Torabi and H. Afshari et al, A DQEM for transverse vibration analysis of multiple cracked on-uniform Timoshenko beams with general boundary conditions, Computers and Mathematics with Applications, 2014, 67, pp.527–541.

[2] Min Yang and Jianhua Zhu , A practical method to calibrate the slant angle of central X-ray for laminography scanning system, NDT and International Journal, 2014, 64, pp.13–20.

[3] S. Chawla, Garg et al, Identification of multiple cracks in a cantilever beam using a new hybrid approach, International Conference, 2014, vol 1, pp.118-123

[4] Ahijit Naik and Pawan Sonawane, Vibration Analysis of a Cracked Beam Using Various Techniques, International Journal of Innovative Research in Science Engineering and Technology, 2014, Vol. 3, pp.17913-17918.

[5] Aniket S. Kamble1 and D. S. Chavan, Identification of crack parameters in a cantilever beam using experimental and wavelet analysis, International Journal on Mechanical Engineering and Robotics (IJMER), 2014 Volume-2, pp.23-27.

[6] P. Yamuna and K.Sambasivarao, Vibration Analysis of Beam With Varying Crack Location, International Journal of Engineering Research and General Science, 2014, Volume 2, pp.1008-1017.

[7] Malay Quila and Prof. Samar Ch. Mondal et al, Free Vibration Analysis of an Un-cracked & Cracked Fixed Beam, IOSR Journal of Mechanical and Civil Engineering , 2014, Volume 11, pp.76-83.

[8] Dr. K. B. Waghulde and Dr. Bimlesh Kumar ,Vibration Analysis of Cracked Cantilever Beam With Suitable Boundary Conditions, IJISET - International Journal of Innovative Science, Engineering & Technology, 2014 Vol.1, pp.2348 – 7968.

[9] Mahmoud Nadim Nahas and Mahmoud Ali Alzahrani, Restoration of Natural Frequency of Cracked Cantilever Beam Using CNT Composite Patch: A Finite Element Study, International Journal of Innovative Science, Engineering & Technology,2013, Vol.1, pp.1-6.

[10] Prathamesh M. Jagdale and Dr. M. A. Chakrabarti, Free Vibration Analysis of Cracked Beam, Int. Journal of Engineering Research and Application, 2013, Vol. 3, pp.1172-1176.

[11] S. P. Pogal and Dr. R. K. Behera, Vibration Analysis of Cracked Beam, International Journal of Advanced Engineering Techanalogy, 2012, Vol.3, pp.371-377.

[12] Yue Cheng and Zhigang Yu , Vibration analysis of a cracked rotating tapered beam using the p-version finite element method, Finite Elements in Analysis and Design, 2011, 47, pp.825–834.

[13] Saidia bdelkrim and Hamouineabdel madjid, Crack detection in concrete beams using experimental modal data, 8th International Conference on Structural Dynamics, 2011, vol 1, pp.2123-2126.

[14] Murat Kisa and M. Arif Gurel, Free vibration analysis of uniform and stepped cracked beams with circular cross sections, International Journal of Engineering Science, 2011, 45, pp.364–380.

[15] RS Prasad, SC Roy and KP Tyagi, Effect of Crack Position along Vibrating Cantilever Beam on Crack Growth Rate, International Journal of Engineering Science and Technology, 2010, Vol. 2, pp.837-839a

